THE USE OF MAGIC IN MATHEMATICS : FROM PRIMARY SCHOOL TO HIGHER EDUCATION

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Abstract

Why use Magic for teaching Mathematics ?

Magicians know that once the surprise has worn off the audience will seek to understand how the trick works. This is particularly true in France. Is it our “esprit cartésien” ?

The aim of every teacher is to interest their students, to provoke their curiosity and a magic trick will bring them to ask **how**? and **why**? and the lesson can begin :

- Surprise the class and provoke the curiosity of the students
- Trying to understand a trick by yourself requires concentration and attentiveness
- The lesson begins quicker than usual : “*today we are going to do a little magic”*
- Learning a trick isn't always easy : you have to work at it.
- The fun aspect of the presentation means it will be memorized more easily and for longer.
- A spectacular example will interest even the student who shows the most aversion to calculus or mathematics, and the ensuing lessons will benefit from this.
- The student who is convinced that (s)he doesn't understand anything to do with Maths and therefore makes no effort, will be shown that (s)he is wrong and that (s)he can improve (for secondary schooling).
- It shows the students that one can teach one subject and yet have other interests, even an art form.
- Whatever the student's professional ambitions, (s)he will be able to see the impact that originality and creativity have when combined with an interest in one's work.
- The pupils or students will go home knowing how to “perform” a magic trick for their family and friends, a trick that they will be able to explain and so enjoy a certain amount of success. Sharing a mathematical demonstration is not easy and that they do so means that they will have worked on, understood and are capable of explaining this knowledge. Isn't this the aim of all teaching?

So, at any level, the teacher begins the lesson with a magic trick rousing interest of the students, then help them to discover how the trick works before the theory behind the trick is finally high-lighted with/by the students themselves.

In this article we will consider only card tricks underpinned by mathematical principles. Martin Gardner, a university lecturer, has written several books and articles on these well-known tricks in the magazine Science. **It is not necessary the teacher has to be a magician because all of presented tricks are self-working magic tricks**.

Keywords - Math, magic, teaching, bijection, gilbreath, nine proof, line equation, modulo, congruent.

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1 INTRODUCTION

From the beginning of time people have feared what they don't understand and sought logical explanations for inexplicable phenomena. To begin with they considered them to be the work of magic, then the work of the gods, then the work of God himself. The church discouraged the spread of the conjurer's art as it preferred not to have rational explanations for what was considered supernatural.

The first magic tricks were performed in the Middle Ages by clowns and/or con-men who would entertain passers-by by getting them to bet on the position of a ball hidden in one of three up-turned goblets, the bet being usually lost. This trick is known now days under the name 'cups and balls'[1]. The first book considered to be about modern Magic (or should we say "conjuring") was written and printed the 16th Century. It was about Magic with ropes[2]. It wasn't until the end of the 19th Century that the word Magic took on its present-day meaning when the famous magician Houdini, father of modern Magic, made it the art it is today.

Since then a good number of principles have been invented and improved on by magicians and gamblers [3], especially for card tricks with bets.Since the 1980s the secrets, that were once passed on from master to apprentice, are now universally available through the use of video cassettes and modern communication technology, and Magic has become a Big Business.

The aim of the magician is to hide the principles he uses (using maths, physics, psychology, slight of hand, etc...) by disguising the trick so that the audience has no way of discovering how it is done thus allowing the Magic to remain.

The teacher can do exactly the opposite : unraveling a Magic trick to high-light the principles used !

We will begin with a trick for the primary school classroom, showing how to use memory techniques to learn the nine times table and explaining why this property can be used to check when we "carry" and work out multiplication using several numbers/digits. We will use this same trick to explain equations to middle school pupils. This magic trick can explain the congruence in high school.

We will then continue with a trick for the secondary school classroom, used to explain first order equations and also the modulo operation.

This trick can also be used at university level to explain injection, surjection and bijection as well as palindromic properties, alternative colors, etc....

We will end by a trick who are the proof that a new mathematical langage give the right results : the COQ, thanks to whom prove some theorems which can't be prooved by the classicum mathematical way.

But let's not reveal all the secrets just yet … !

2 PRIMARY SCHOOL : TO EXPLAIN THE 9 PROOF, THE DIFFERENCE BETWEEN DIGIT AND NUMBER

2.1 The trick exposed by the magician (teatcher)

To simplify the explanation, let us asume that back cards is red, that the interesting position is the fitht (thus n=5) and that the value of this 5^{th} card is the 3 of Club.

A regular 52-card deck, a envelope (in which a prediction will be inserted), a Little peace of paper (let us say A5 or A6) and a pen is requiered to realice this trick.

Take out cards and ask to a pupil to shuffle and cut as many times he wants. Look at the card in n^{th} positionfrom the top when the deck is face down (the red back are visible). Take then the paper and writte down, thanks to your pen, the value and the color of the sawn card (le tus asume it is the 3 of Club !). Fold the paper and insert it into the envelope, which has been shown empty before.

Say to the pupils' assembly "Each of you choose a number different from 0 (it is easier for you to choose a one digit number). Multiply it by 3. Add 6 to the result. Multiply this last result by 3 one more time. You obtain necessarily a number (say 125 ; the 3 digits are 1, 2 and 5 which are added together to obtain 1+2+5=8). Do the same operation with your final number (after calculation !) until you got aonly a one digit number. Finally soustract 4" (always the result is 9. We want the 5th card here ; that's why 4 must be soustract !).

Ask to one of pupils to say to the other its final calculated number (He must say 5 !). Count one by one n cards (here n=5). Put aside the last counted card. Give the envelope to a another pupil, say to him to take outside the paper and to read it aloud. Then, give to the pupil's reader the choosen card in order he have to show the card to everybody … estaunishment and then applauses!

There are many variants of this card trick … The best one will be yours ![1]

2.2 The trick seen by the spectators (pupils)

A spectator shuffle the cards and cuts the deck as many times he wants. The Magician joint sown the prediction on a piece of paper which is putted inside an empty envelop. The spectaeur choose any number (without limitation), does some arithmetic operations in order to obtain a final number. The card postionned at this position is putted away. The prediction is readed and … it matches !

2.3 The mathematical notions used by the trick

The used mathematical notions by this trick are written below :

- The difference between "figure/digit" and "number"
- The three basic operations: the addition, the subtraction and the multiplication
- The 9-proof.

The trick uses the both theorems explained §6.4A and §6.4B. If we multiply a number by 9, automatically the sum of its digits is 9 or is a multiple of 9. The suit is obvious because the magician has converted an unknown number, by everybody except the spectator, into an, only by him, known number whose value is 9 !

2.4 How to go over from the trick to the mathematical explanations?

The explanations can be given in due time :

A. First step - Do the magic trick again bu t:

- every pupil has to choose a number. The whole pupils'number are joint at the blackboard,
- the teacher do the arithmetic operations for a part of the numbers ,
- when the teacher adds the digits of each calculated numbers, he can presents clearly the difference between digit and number. After showing this to the pupils, they could do the same thing with their own calculated number,
- the teacher can work, at this moment, on the numeric order of number !
- the teacher can "prove" that, any number the pupil has choosen, after calculation the number is always 9 !

B. Second step

The teacher get ready to do one more time the trick then he says 'NO'. The pupils must try to find how the initial number has been multiplied by 9 ….!

C. Last step

With the nine property, the teacher can explain the nine-proof …

[1] Please send to me your ideas if you want at magie.carte@ aposte.net

3 SECONDARY SCHOOL : TO EXPLAIN THE 'SIMPLE' FACTORIZATION AND TO EQUATE A PROBLEM

3.1 The trick exposed by the magician (teatcher)

The used magic trick is the same one which is used §2.1.

3.2 The trick seen by the spectators (pupils)

The used magic trick is the same same one which is used §2.1.

3.3 The mathematical notions used by the trick

The mathematical used notions are the same as the previous one. But in fact not completely … it depends on the explanation which the teacher are going to give to the pupils !

3.4 How to go over from the trick to the mathematical explanations?

Here, the interesting mathematical notions are no more the explained ones previously, but trully to equate a problem and to explain how the polynomial developement and factorizeation works. To do it, let us propose the magician point of view, that is :

the chosen number is really unknown		
this number is multiplied by 3	= >	we obtain 3x
6 is added to this calculated number	= >	3x + 6
finally this last result is once again multiplied by 3	= >	3.(3x+6) = 9x+18
factorize by 9	=>	9(x+2)

The magician has done multiply by 9, not the spectator's number, buta n another one … if it is necessary, the teacher can expose the previous explanation (§2.4) if necessary.

4 END OF SECONDARY SCHOOL: TO EXPLAIN THE PARAMETRIC SIMPLE EQUATIONS OF THE FIRST ORDER, TO EQUATE A PROBLEM AND THE NOTION OF MODULO.

4.1 The trick exposed by the magician (teatcher)

Take a new deck and take out the cards. Put away the extra cards as well as the jokers. Fan the cards, backs are visible, in front of a student who thus chooses one of them. While he looks at his card, look the card before in the deck. Memorize its color and its value (let us say the 7 of Heart).

Calculate the position of the 7 of Heart thanks to one of following both equations (eq. 1 or eq. 2). The 7 of Heart position is the 45^{th}. Add 1 to this position to find the position of the selected card (here the 46^{th}).

Find the color and the value of the selected card (here 46^{th} card in the deck). The color is calculated thanks to table 4 and its and the value thanks to the equations (eq. 3 or eq. 4). After calculation, we obtain the 6 of Heart. Announce laut the value and the color of the selected card thanks the best magical way for you … (if you wish, you can untidily mix the deck during your announcement in order to hide the way you have done the trick !

4.2 The trick seen by the spectators (pupils)

A new deck is taken out. One card is freely selected by a spectator. The magician find the identity of the selected card without looking at the back of the card and without touching the card !

4.3 The mathematical notions used by the trick

The used mathematical notions by this trick are written below :

The simple equations defined by parts(parties) and their resolution

The notion of modulo

4.4 How to go over from the trick to the mathematical explanations?

When a card deck is opened, the cards are always in the same order as follows :

First card *Ace of Spade*, second card *2 of Spade*, …, 13th card *King of Spade*, 14th card *Ace of Diamonds*, 15th card *2 of Diamonds*, …, 25th card *Queen of Diamonds*, 26th card *King of Diamonds*, 27th card *King of Club*, 28th card *Queen of Club*, …, 38th card *2 of Club*, 39th card *Ace of Club*, 40th card *King of Heart*, 41th card *Queen of Heart*, …, 51th card *2 of Heart*, 52th card *Ace of Heart*.

The aim of the card trick is :

First to find the card position in the deck knowing the value and the color of the card.

Thus to find the color and the value of the card knowing its position in the deck.

Depends of the case, two or one d'inconnues have to be calculated :

First case : known value : the position ; unknown values : card color and value.

Second case : unknown value : the position ; known values : card color and value.

Let us assume that :

p the card position in the deck,

c the card color.

v the card value.

Thanks to the Table 1 and Table 2, the position of each card value, in the color sentence, is known

Table 1. Card position for the Spades and the Diamondss.

Value	AS	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Position	1	2	3	4	5	6	7	8	9	10	11	12	13

So the jacks of Spade and Diamonds are at the eleventh position in its repective color sentence.

Table 2. Card position for the Clubs and the Hearts.

Value	AS	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Position	13	12	11	10	9	8	7	6	5	4	3	2	1

So the jacks of Club and Heart are at the third position in its repective color sentence.

To know the card position in the complete deck, the only missing information is the number of color is past. The Table 3 expose the position of the jack in the entire deck.

Table 3. Jack positon depending of the its color

Couleur avant	0	1	2	3
Jack position	11 = 11	24 = 13 + 11	29 = 2x13 + 3	42 = 3x13 + 3

For the color, it is necessary to associate, the same as for the card values, the color to a digit. I suggest giving the value 0 to the first color (because there is no color before), 1 to the second color, 2 to the third and finally 3 to the fourth

The result is in our example:

Table 4. Card position depending of the colors – multiplicative coefficient depending of the color.

Color name	Spade (c=0)	Diamond (c=1)	Club (c=2)	Heart (c=3)
Cards postion	1st to 13th	14th and 26th	27th et 39th	40 et 52

So we obtain the following equation for the card position in the deck :

p = c*13 + v if c=0 or c=1 (for Spade or Diamond) **(Eq. 1)**

p = c*13 + (13-v+1) if c=2 or c=3 (for Club or Heart) **(Eq. 2)**

The graphic representation of these functions is given in fig. 1

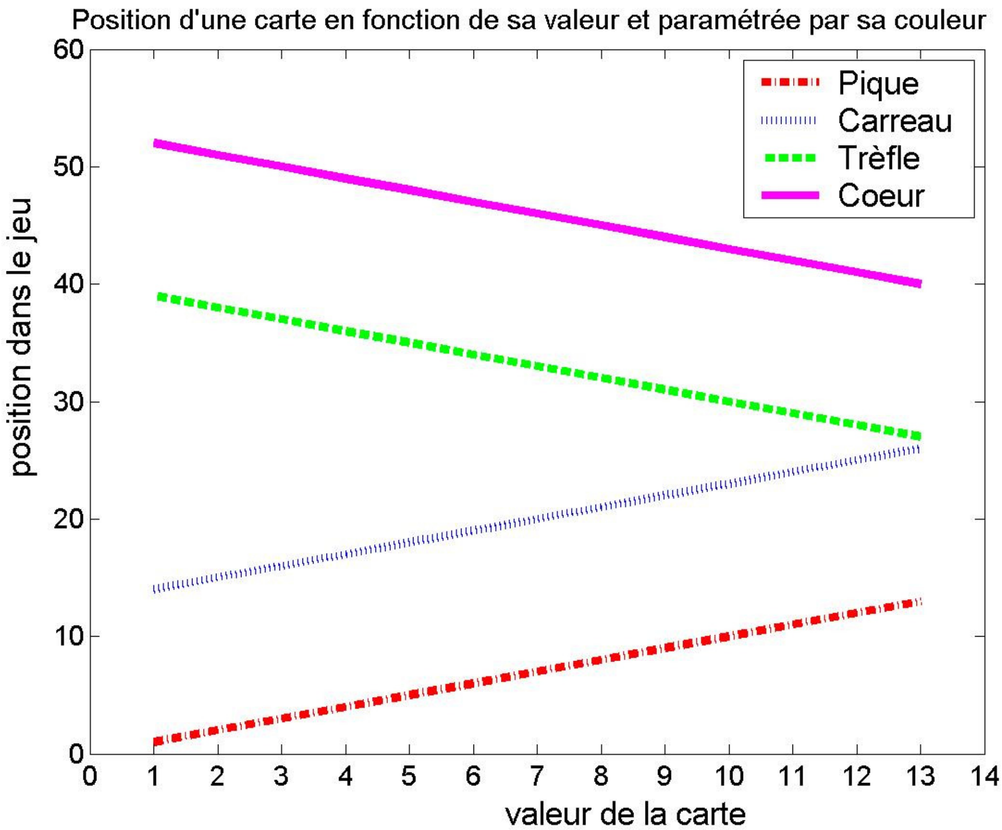

Fig. 1. Card position according to its value and parametrized by its color.

Remark :

Thanks to table 4, the card position is between the 1st and the 13th because it is Spade. Thanks to table 1, the card position of the value 5 is the 5th , the 18th . Thanks to table 2, the card position of the value 5 is 35th and 48th. Thus its position is parametrized by its color.

So we obtain the following equation for the card value in the deck :

v = p - c*13 if c=0 or c=1 (for Spade or Diamond) **(Eq. 3)**

v = c*13 + (13-p+1) if c=2 or c=3 (for Club or Heart) **(Eq. 4)**

The graphic representation of these functions is given to the fig. 2

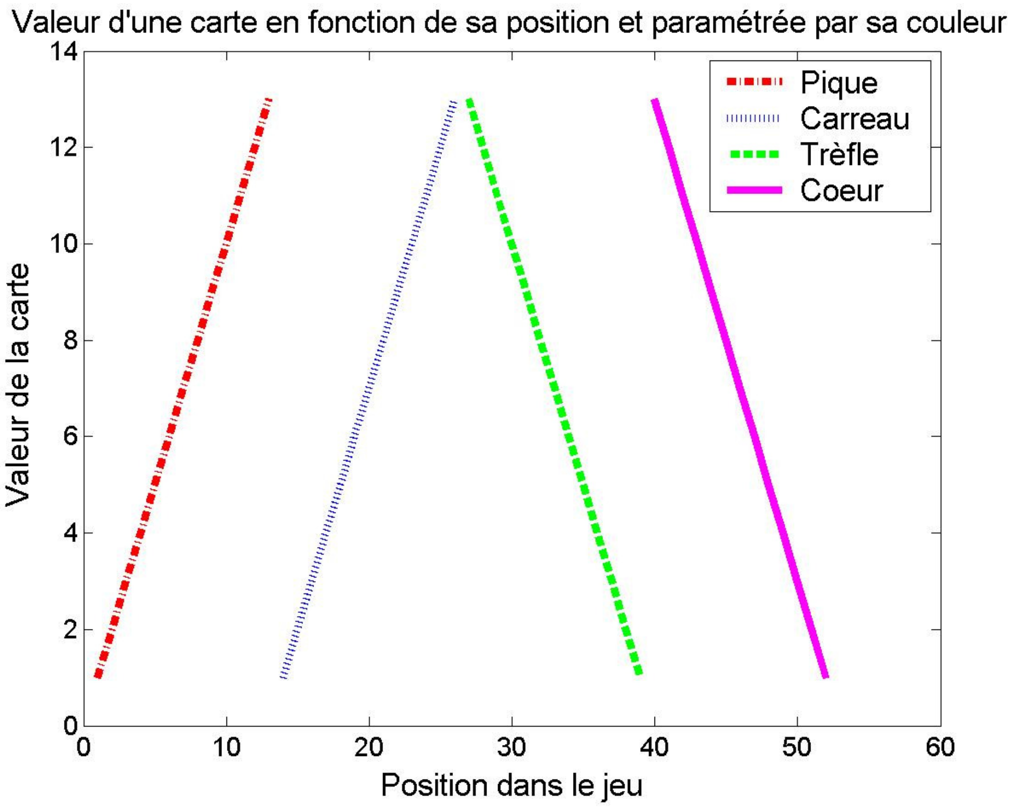

Fig. 2. Card value according to its position and parametrized by its color.

The equations are yet written. Let us do Magic !

A. *The trick : the value and the color of the card are given and the magician finds the card*

We ask to the spectator to think about a card. Then it is requiered to him to name it. (For example eight of Spade). The Magician cut the deck and the top card is the thinking card !

Spade	=> 0 thus 0*13 =	0
8 value =>		8

The card position in the deck is the		8^{th}

B. *The trick : the card position is given and the magician find the card identity*

The given position is the 37^{th} :

In order to find the place soustract 13 to the card position until finding a calculated number between 1 and 13 (each color has 13 cards !)

- First calculation 37 – 13 = 24 – 24 is not between 1 and 13. A new calculation is required.
- Second calculation : 24 – 13 = 11. That's over. The card value is found

Finding the color :

- Spectator's number between 1 and 13 => The color is Spade.
- Spectator's number between 14 and 26 => The color is Diamonds.
- Spectator's number between 27 and 39 => The color is Club.
- Spectator's number between 40 and 52 => The color is Heart.

How does it can be explain with mathematical notions ?

The division by 13 is used (last class of primary school level) with the notion of quotient and rest : 37=2*13+11 (n = qp+r ; with q as the quotient and r as the rest)

The notion of modulo (middle class in the secondary school) : 37 = 11 modulo [13]

C. to go easyer or to go further

To simplify things, the card position of the third and the fourth color must be inverted. Like that each color begins by a King and ends by an Ace. The new order of the deck is describing below :

First card *Ace of Spade*, second card *2 of Spade*, …, 13th card *King of Spade*, 14th card *Ace of Diamonds*, 15th card *2 of Diamonds*, …, 25th card *Queen of Diamonds*, 26th card *King of Diamonds*, 27th card Ace *of Club*, 28th card 2 *of Club*, …, 38th card *Queen of Club*, 39th card King *of Club*, 40th card Ace *of Heart*, 41th card 2 *of Heart*, …, 51th card *Queen of Heart*, 52th card King *of Heart*.

With this card deck order, the card positions are given only by table 1 and eq. 1 !

To complícate the spectators' understanding of the trick, the cards can be quiet ordered : ordered cards is called a chapelet[2]

5 HIGH SCHOOL : EXPLAIN THE INJECTION, SURJECTION AND BIJECTION

5.1 The trick exposed by the magician (teatcher)

The used magic trick is the same one which is used §4.1, §4.4A and §4.4B.

5.2 The trick seen by the spectators (students)

The used magic trick is the same one which is used §4.1, §4.4A and §4.4B.

5.3 The mathematical notions used by the trick

The mathematical used notions are the same as the previous one. But in fact not completely … it depends on the explanation which the teacher are going to give to the students !

5.4 How to go over from the trick to the mathematical explanations?

Here, what interests us, they are not the properties written previously but well and truly the notions of groups with finite elements, the notion of injection, surjection and finally bijection. To do it, I propose the point of view of the magician, that is :

A. Mathematical explanation of injection

Theorem : Let suppose F is a function between two sets. If each element of the departure set has at least one element (called 'image') in the arrival set, the function is injective.

Let us suppose " the selected card is a 5 ". This card could be at one of four following position : the 5th, 18th, 35th, 48th. For each card value (from Ace to King), the same property exists. All possibilities are shown in fig. 3

B. Mathematical explanation of surjection

Theorem : Let suppose F is a function between two sets. If each element of the arrival set has at least one element (called 'antecedant') in the departure set, the function is surjective.

Let us suppose " the card position is the 25th ", or let us suppose " the card position is the 38th " , the card value is always the Queen. For each card position (from 1st to 52th), the same property exists. All possibilities are shown in fig. 4

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[2] Mettre une référence sur l'utilisation des chapelets

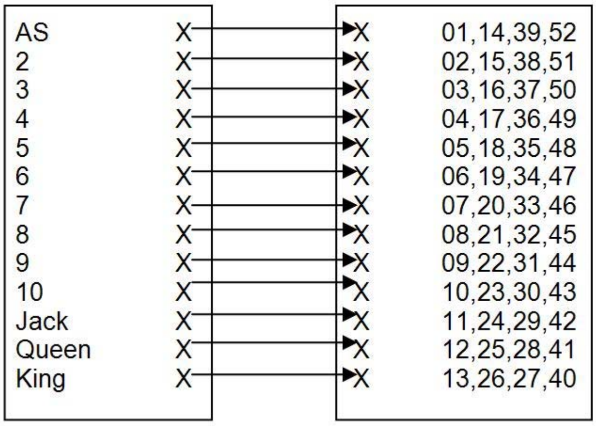

Fig. 3. Injection : departure set is the card value and the arrival set is the card position in the deck.

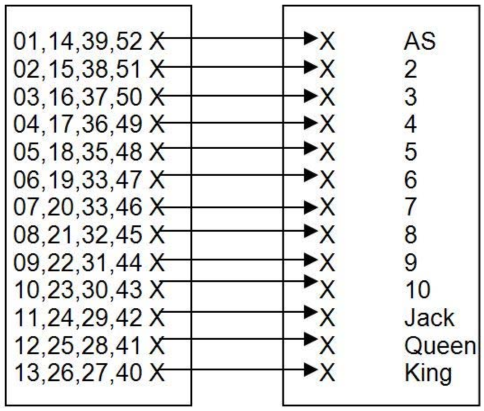

Fig. 4. Surjection : departure set is the card position in the deck and the arrival set is the card value.

A another way to construct a surjection is to bind the card position with the card color. Let us suppose " the card position is the 25th ", or let us suppose " the card position is the 15th " , the card color is always the Diamonds. For each card position (from 1st to 52th), the same property exists. All possibilities are shown in fig. 5.

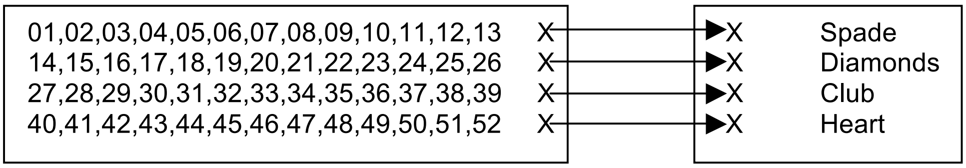

Fig. 5. Surjection : departure set is the card position in the deck and the arrival set is the card color.

C. Mathematical explanation of bijection.

Theorem : Let us suppose F is a function between two sets. If F is at the same time injectiv and surjective, F is thus bijective.

The departure set is the card position in the deck and the arrival set is the caard identity (value and color). F, the function between these both sets is a bijection !

6 HIGH SCHOOL : EXPLAIN THE CONGRUENCE

6.1 The trick exposed by the magician (teatcher)

The used magic trick is the same one which is used §4.1, §4.4A and §4.4B.

The spectator must not any more to calculate a suit of multiplication and addition. He chooses a number (whatever he wants !). He sums the digits of its number which is soustracted to the selected number. The spectator is requiered to ad done more time the digits of the calculated number (and do the same thing) until he obtains a digit. The magicians goeson like the presented trick before.

6.2 The trick seen by the spectators (students)

The used magic trick is the same one which is used §4.1, §4.4A and §4.4B.

6.3 The mathematical notions used by the trick

The mathematical used notions are the same as the previous one. But in fact not completely … it depends on the explanation which the teacher are going to give to the students !

6.4 How to go over from the trick to the mathematical explanations?

Here, two mathematical properties are used. They do not seem necessarily evident :

A. First theorem

Theorem : If a natural number, higher than 10, is written on the 10-basis. Soustract the sum of its digits to the first number. The result is a natural number which can be divided by 9.

This mathematical property is written in the routine called 'Digital roots' [4] (pp 164-165). The demonstration can be then asked to the students.

Let us study the congruences of the powers of 10 modulo 9. We obtain:

$$\forall k \,,\, 10^k \equiv 1 \; [9] \qquad \textbf{(Eq. 5)}$$

Let us write the number n on the basis of 10^k. We obtain:

$$n = \sum_{k=0}^{N} \alpha_k \, 10^k \qquad \textbf{(Eq. 6)}$$

Let us write then the congruence of n. We obtain:

$$n \equiv \sum_{k=0}^{N} \alpha_k \; [9] \qquad \textbf{(Eq. 7)}$$

Let us soustract the sum of the digits on both sides of the equation. We obtain:

$$n - \sum_{k=0}^{N} \alpha_k \equiv \sum_{k=0}^{N} \alpha_k - \sum_{k=0}^{N} \alpha_k \; [9] \qquad \textbf{(Eq. 8)}$$

$$n - \sum_{k=0}^{N} \alpha_k \equiv 0 \; [9] \qquad \textbf{(Eq. 9)}$$

This last equation proves that the number can be divided by 9 !

This mathematical property was used many times by the magicians. We can cite Fulves, K. for his routine called ' Familliar Spirit ' [5]

B. Second theorem

Theorem : If a natural number, which can be divided by 9, is written on the 10-basis, the sum of its digits is a natural number which can be too divided by 9.

For exemple, le tus choose a natural number between 9 and 81, written on the 10-basis. The sum of its digits is always 9.

This mathematical property is written in the routine called 'Persistant root' [4] (pp 165-166). This mathematical property was used many times by the magicians. We can cite Lorayne, H. in his book [6] (pp196-197).

Let us write the number n on the basis of 10^k. We obtain:

$$n = \sum_{k=0}^{N} \alpha_k \, 10^k \qquad \textbf{(Eq. 10)}$$

Let us write then the congruence of n. We obtain:

$$n \equiv \sum_{k=0}^{N} \alpha_k \; [9] \qquad \textbf{(Eq. 11)}$$

Let us write that n can be divided by 9. We obtain:

$$n \equiv 0 \qquad \textbf{(Eq. 12)}$$

Thanks to the both last equations, we obtain:

$$\sum_{k=0}^{N} \alpha_k \equiv 0\ [9] \qquad \textbf{(Eq. 13)}$$

The last equation proves that the sum of its digits can be divided by 9.

C. Generalization

We can even go further generalizating these both theorems.

Theorem 1 : Let us suppose $10^k \equiv 1\ [m]$ and let us write $\beta_0, \beta_1, \ldots, \beta_{k-1}$ the residues of 10^j modulo m in order of appearance. n is a natural numberr, greather or equal than m+1, written in 10-basis. Soustract the ponderated sum of its digits $\left(\sum_{p=0}^{N}\sum_{i=0}^{k-1}\beta_i\alpha_{i+kp}\right)$ to the first number. The result is a natural number which can be divided by m.

Theorem 2 : Let us suppose $10^k \equiv 1\ [m]$ and let us write $\beta_0, \beta_1, \ldots, \beta_{k-1}$ the residues of 10^j modulo m in order of appearance. If a natural number, which can be divided by m, is written on the 10-basis, the ponderated sum of its digits $\left(\sum_{p=0}^{N}\sum_{i=0}^{k-1}\beta_i\alpha_{i+kp}\right)$ is a natural number which can be too divided by m.

Idea of demonstration: write previous both demonstrations by replacing the integer n as follows :

$$n = \sum_{p=0}^{N} 10^{kp}\left(\sum_{i=0}^{k-1}\alpha_{i+kp}10^i\right) \qquad \textbf{(Eq. 14)}$$

For example, let us take the successive residues of 10^j modulo 7:

$$\begin{aligned}
\beta_0 &= 10^0 \equiv 1\ [7]\\
\beta_1 &= 10^1 \equiv 3\ [7]\\
\beta_2 &= 10^2 \equiv -1\ ou\ 6\ [7]\\
\beta_3 &= 10^3 \equiv -3\ ou\ 4\ [7]\\
\beta_4 &= 10^4 \equiv 1\ [7]
\end{aligned} \qquad \textbf{(Eq. 15)}$$

7 SUPÉRIEUR : LA RECHERCHE

Mr G. Huet[3] *"presents the full axiomatisation and proff development of a non-trivial property of binary sequences, inspired from a card trick of N. Gilbreath[7, 8, 9]. This case study illustrates the power and naturalness of the Calculus of Inductive Constructions as a specification language, and outlines a uniform methodology for conducting inductive proofs in the Coq proof assistant."* [10].

I contacted Mr. Huet to know his motivations to use a magic trick as example. He answered me by email[4]: " Nicolas de Bruijn who ate at home[5] in 90's did a card trick to my children and when I realized that this trick was exact I chose this trick as example of proof by recurrence. During many years, this

[3] His CV on internet at http://yquem.inri'.fr/~huet/ He is member or french science academy '
(http://www.academie-sciences.fr/index.htm). '

[4] Em' il send the 28th M'y 2009 to P. Schott 'at 'schott@esiea.fr' '

[5] Mr Huet is the writer of the email. '

proof was my standard demo of Coq language, and I had made a challenge for automatic demo systems.

Let us note that Mr G. de Bruijn, who has shown to the autor the magic trick, has used this argument to prove a crystallography result [11].

8 CONCLUSION, GOING FURTHER AND PROSPECTS

8.1 Conclusion

So, at any level, the teacher begins the lesson with a magic trick rousing interest of the students, then help them to discover how the trick works before the theory behind the trick is finally high-lighted with/by the students themselves. It is not necessary the teacher has to be a magician. In fact, all of presented tricks are self-working magic tricks !

When the teacher has shown the magic trick, he has two different ways to teach : the downward method and the ascending method.

The downward method (the professor teaches the student) usually used in France, can be used by doing one more time the magic trick and at each step, the teacher explain the mathematical notions.

The ascending method (it is the student who will have to explain to the professor), can be used by proposing that the students do the magic and try to explain how it works. Of course the teacher is here like a guide in the way of knowledge. The ascending method has the merit of showing whether the posed problem is well understood. As wrote Boileau [12] " what conceives well expresses itself clearly, and the words to say it arrive easily. "[6,7].

8.2 Going further

Many notions exists in matematics. For all notions in the mathematics program from primary school to higher education, do card magic trick exist ? I don't think so ! But it could be a way of teaching that the students imagine a magic trick for a explained notions by the teacher …

8.3 Prospects

I use magic as a vector of mathematical informations to teach. We can think that another matter could be teached by magic ! For exemple optics [13], automatism, electromagnetic fields, and so on ... A project with math, informatic and … magic could be proposed to the students in high school to prove by classic way the Gilbreath's principle and to writte a program in C with a java interface in order to show this trick with the computer on the net[8].

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